Journal of the Korean Physical Society, Vol. 39, No. 6, December 2001, pp. L0∼L0 Letters

Spectral Functions for Doped Carbon Nanotubes using the Gross-Neveu Model

Changrim AHN* and Eunkyung PAIK

Department of Physics, Ewha Womans University, Seoul 120-750

(Received 30 August 2001)

We use integrable and conformal quantum field theories to compute the single-particle spectral function of armchair-type carbon nanotubes away from half-filling. The gapless charge sector is described by the Luttinger model and is solved by using conformal field theory. The other sectors are known to possess $SO(6)$ symmetry at low temperature and are described by the $SO(6)$ Gross-Neveu model. The correlation function of this massive sector is computed using exact form factors. Due to intermediate particle states, the spectral function is exactly determined by the given form factors between certain threshold energies. We derive an exact expression explicitly for certain values of the Luttinger parameters at low frequency to illustrate the power of this approach.

I. INTRODUCTION

Low-dimensional quantum field theories (QFTs) are actively applied to various strongly interacting condensed matter systems. The main advantages of this approach are as follows: the continuum theories can describe the universality class of the lattice models which restrict the microscopic dynamics of strongly correlated electrons to specific form of lattice interactions. The universality attained by the QFTs, therefore, can show physical properties independent of microscopic models. Another advantage is their analytic computational power. The perturbative solutions of the QFTs have been used to understand the behavior of the systems from high to low temperatures.

However, this QFT approach has a fundamental difficulty for applications to a strongly correlated system because the perturbative method fails in these cases due to the strong interactions. Even numerical analysis is not easy for the continuum theories. Therefore, this area can be a very interesting theoretical laboratory where non-perturbative QFTs can be tested and applied with connection to experiments. Up to now truly non-perturbative QFTs have been realized only in $(1+1)$-dimensional space-time. In this dimensionality, there are certain classes of the QFTs, called integrable QFTs, whose scattering amplitudes are exactly known. In addition, very efficient theoretical tools which can extract physical quantities out of the S-matrices have been developed. The form factor approach is one of these and can generate the correlation functions to high accuracy and sometimes exactly in momentum space.

*E-mail: ahn@dante.ewha.ac.kr

The low dimensionality which the integrable QFTs are restricted to is not a major huddle any more thanks to recent progress in experimental technologies. Many new and interesting materials can be described effectively as one-dimensional objects. Single-walled carbon nanotubes are a typical example [1]. In addition to various potential applications and theoretical interests [2], these materials show Mott insulator properties and can be described by an interesting theoretical model, namely, the two-leg Hubbad ladder. In this model, electrons can hop in inter- and intra-chains and have on-site Coulomb interactions. It has been noticed that armchair (N,N) nanotubes can be mapped into Hubbard ladders with the effective ladder coupling [3,4]

$$u_{\rm eff} = \frac{u}{N} \tag{1}$$

where u is a (strong) on-site Coulomb interaction of the nanotubes. Therefore, the effective QFTs for strongly interacting carbon nanotubes can be obtained by using the perturbative computations for Hubbard ladders. Another interesting feature is that the Hubbard ladders show a phase diagrams very similar to those of cuprate high T_C superconductors. In particular, the Mott insulator phase of the Hubbard ladders arising from the Coulomb repulsion can describe qualitatively the controversial pseudo-gap regions in a phase diagram characterized by both spin and charge gaps, as well as d-wave pairing.

In this paper, we will compute exact single-particle spectral function of doped carbon nanotubes by using the $SO(6)$ Gross-Neveu (GN) model [5], a well-known integrable QFT, along with the Luttinger model for a gapless charge sector. The non-perturbative S-matrix and the particle spectrum can be used to compute the

form factors and the correlation function. In particular, we derive an explicit spectral function at low frequency, which is of most interest, and point out the threshold effects arising from the particle spectrum of the GN model. Our result shows a qualitative difference from the case at half-filling described by the $SO(8)$ GN model [6,7].

II. TWO-LEG HUBBARD LADDER AT HALF-FILLING

We begin with non-interacting electrons hopping on a two-leg ladder described by the Hamiltonian

$$\begin{aligned} H_0 = & - \sum_{x,\sigma} \Big\{ t a^\dagger_{1\sigma}(x+1) a_{1\sigma}(x) + t a^\dagger_{2\sigma}(x+1) a_{2\sigma}(x) \\ & + t_\perp a^\dagger_{1\sigma}(x) a_{2\sigma}(x) + \text{h.c.} \Big\}, \end{aligned} \tag{2}$$

where a_l ($a_l^\dagger$) is an electron annihilation (creation) operator on leg l ($l = 1, 2$) of the ladder, x is a discrete coordinate along the ladder and, $\sigma = \uparrow, \downarrow$ is a spin index. The parameters t and $t_\perp$ are hopping amplitudes along and between the ladder's rungs.

Since the effective interactions become weak, as mentioned in Eq. (1), one can diagonalize the unperturbed Hamiltonian H_0 by bonding/antibonding pairs:

$$c_{j\sigma} = \frac{1}{\sqrt{2}}(a_{1\sigma} + (-1)^j a_{2\sigma}). \tag{3}$$

At half-filling with one electron per site, particle-hole symmetry requires that the Fermi velocities, v_{Fj}, of the two bands $j = 1, 2$ be equal. For the low-energy behavior, we can linearize the $c_{j\sigma}$'s near the Fermi surface k_{Fj}:

$$c_{j\sigma} \sim c_{Rj\sigma} e^{ik_{Fj}x} + c_{Lj\sigma} e^{-ik_{Fj}x}, \tag{4}$$

where R and L correspond to right and left moving modes about the Fermi surface. In terms of these operators, the H_0 becomes

$$H_0 = v_F \int dx \sum_{j\sigma} [c^\dagger_{Rj\sigma} i\partial_x c_{Rj\sigma} - c^\dagger_{Lj\sigma} i\partial_x c_{Lj\sigma}]. \tag{5}$$

The next step is to bosonize the c's:

$$c_{Pj\sigma} = \kappa_{Pj\sigma} e^{i\phi_{Pj\sigma}}, \quad \text{P} = +, - = \text{R}, \text{L}. \tag{6}$$

Here, $\kappa_{Pj\sigma}$ are the Klein factors.

To separate charge and spin, one can define new sets of bosons:

$$\begin{aligned} \phi_{P1} &= \frac{1}{2}(\phi_{P1\uparrow} + \phi_{P1\downarrow} + \phi_{P2\uparrow} + \phi_{P2\downarrow}), \\ \phi_{P2} &= \frac{1}{2}(\phi_{P1\uparrow} - \phi_{P1\downarrow} + \phi_{P2\uparrow} - \phi_{P2\downarrow}), \\ \phi_{P3} &= \frac{1}{2}(\phi_{P1\uparrow} - \phi_{P1\downarrow} - \phi_{P2\uparrow} + \phi_{P2\downarrow}), \\ \phi_{P4} &= \frac{P}{2}(\phi_{P1\uparrow} + \phi_{P1\downarrow} - \phi_{P2\uparrow} - \phi_{P2\downarrow}). \end{aligned} \tag{7}$$

If one further refermionizes these new bosons,

$$\Psi_{Pa} = \kappa_{Pa} e^{i\phi_{Pa}}, \quad a = 1, 2, 3, \tag{8}$$

$$\Psi_{P4} = P\kappa_{P4} e^{i\phi_{P4}}, \tag{9}$$

and introduces Majorana fermions defined by

$$\Psi_{aP} = \frac{1}{\sqrt{2}}(\psi_{2a,P} + i\psi_{2a-1,P}), \qquad (a = 1, ..., 4), \tag{10}$$

one can write the free Hamiltonian as follows:

$$H_0 = \int dx \sum_{a=1}^{8} \left(\psi^\dagger_{Ra} i\partial_x \psi_{Ra} - \psi^\dagger_{La} i\partial_x \psi_{La} \right). \tag{11}$$

The interactions between electrons, in general, break the large symmetry of the free fermions down to $U(1) \times SU(2)$, *i.e.*, total charge and spin conservation. Since we are interested in low-energy physics, we can keep only the marginal interactions, namely, the four-fermion interactions. It turns out that there are nine four-fermion terms which preserve charge and spin. It is remarkable that an one-loop renormalization group (RG) analysis show that the nine coupling constants converge into a fixed ray in the infrared (low energy) limit [6]. Therefore, the low-energy effective QFT for the carbon nanotubes is the $SO(8)$ GN model,

$$H_{int} = g \sum_{1 \le b < a \le 8} G^{ab}_R G^{ab}_L, \qquad G^{ab}_P = i\psi_{a,P}\psi_{b,P}. \tag{12}$$

III. AWAY FROM HALF-FILLING

Most real physical systems have some kind of disorder or impurity. Being Mott insulators, the carbon nanotubes become conductors when doped because the $U(1)$ charge sector becomes gapless. Important interaction effects arise from processes where both the two incoming and the two outgoing particles are all at the Fermi energy. The Hamiltonian density of the interaction then is

$$\begin{aligned} h_{int} &= g_{1,abcd} \psi^\dagger_{Rsa} \psi^\dagger_{Ltb} \psi_{Rtc} \psi_{Lsd} \\ &+ g_{2,abcd} \psi^\dagger_{Rsa} \psi^\dagger_{Ltb} \psi_{Ltc} \psi_{Rsd}. \end{aligned} \tag{13}$$

The first and the second terms describe backward (g_1) and forward (g_2) scattering between particles at the Fermi surface, respectively. Away from half-filling, no Umklapp terms appear. Here, the band indices a, b, c, and d describe either bonding (0) or anti-bonding (π) combinations of the Hubbard ladders. Summation over the band and the spin indices, s and t, is understood in Eq. (13). The interactions allowed by momentum and energy conservation are

$$\begin{aligned} g_{1,0000} &= g_{1,\pi\pi\pi\pi} \equiv g_{11}, \; g_{1,00\pi\pi} = g_{1,\pi\pi00} \equiv g_{12}, \\ g_{1,0\pi0\pi} &= g_{1,\pi0\pi0} \equiv g_{13}, \; g_{2,0000} = g_{2,\pi\pi\pi\pi} \equiv g_{21}, \\ g_{2,0\pi\pi0} &= g_{2,\pi00\pi} \equiv g_{22}, \; g_{2,00\pi\pi} = g_{2,\pi\pi00} \equiv g_{23}. \end{aligned}$$

The big $SO(8)$ symmetry at half-filling in the low-energy limit is broken to a smaller, but still big, symmetry, namely, $U(1) \times SO(6)$. The low-energy effective QFT can be expressed in terms of the bosons introduced in (7) [8]. The non-interacting Hamiltonian is the Luttinger liquid model

$$H_0 = \sum_{a=1}^{4} H_a, \tag{14}$$

where

$$H_a = \frac{\pi v_F}{2} \int dx \left[(1+M_a)\,\Pi_a^2 + \frac{1}{\pi^2}\,(1-M_a)\,(\partial_x \varphi_a)^2 \right], \tag{15}$$

and the Luttinger parameters are given by

$$M_{2,3} = (g_{11} \pm g_{13})/2\pi v_F,\ M_{1,4} = M_{2,3} - (g_{21} \pm g_{22})/\pi v_F. \tag{16}$$

The interaction Hamiltonian becomes

$$H_{int} = \frac{1}{\pi^2} \int dx \left[(g_{23} - g_{12}) \cos 2\vartheta_4 \cos 2\vartheta_3 + g_{23} \cos 2\vartheta_4 \cos 2\varphi_3 + g_{11} \cos 2\varphi_2 \cos 2\varphi_3 + g_{12} \cos 2\vartheta_4 \cos 2\varphi_2 - g_{13} \cos 2\varphi_2 \cos 2\vartheta_3 \right], \tag{17}$$

where the ϑ_a's are the dual fields to φ_a defined by

$$\Pi_a = \frac{1}{\pi} \partial_x \vartheta_a. \tag{18}$$

These fields are related to the bosons defined previously by

$$\varphi_a = \phi_{Ra} + \phi_{La}, \qquad \vartheta_a = \phi_{Ra} - \phi_{La}. \tag{19}$$

According to the RG computations [8], the phase diagram is divided by two critical lines $g_{1i} = 0$ and $g_{1i} = 2g_{2i}$. In particular, in the "d-wave superconductor phase" where $0 < g_{1i} < 2g_{2i}$, a numerical investigation and one-loop RG equations show that the parameters take the asymptotic form

$$g_{ij}(l) = \frac{g_{ij}^0}{l_c - l} \tag{20}$$

with

$$-g_{11}^0 = g_{12}^0 = g_{23}^0 = \frac{1}{4},$$
$$g_{13}^0 = 0, \quad g_{21}^0 = -\frac{3}{16}, \quad g_{22}^0 = \frac{1}{16}.$$

One can notice from Eq. (16) that $M_2 = M_3 = -M_4$. If one defines $\vartheta_4 = \tilde{\varphi}_4$ and $\varphi_4 = \tilde{\vartheta}_4$, one finds that the three Luttinger parameters become identical and that, in the infrared limit, the interacting Hamiltonian becomes

$$H_{int} = \frac{g_{11}}{\pi^2} \int dx \left[\cos 2\tilde{\varphi}_4 \cos 2\varphi_3 + \cos 2\varphi_2 \cos 2\varphi_3 + \cos 2\tilde{\varphi}_4 \cos 2\varphi_2 \right]. \tag{21}$$

In terms of the Majorana fermions introduced in Eqs. (8) and (10), this corresponds to the $SO(6)$ GN model along with the $U(1)$ Luttinger model for the charge degrees of freedom φ_1, ϑ_1. Therefore, the effective QFT of the doped Hubbard ladder is given by

$$H = H_1 + H_0 + g \sum_{3 \le b < a \le 8} G_R^{ab} G_L^{ab}. \tag{22}$$

The GN model is an integrable QFT with interacting Majorana fermions with a dimensionless coupling constant g. Dynamical symmetry breaking produces the fermion mass while preserving an infinite number of conservation laws [9]. The on-shell particle spectrum of the $SO(6)$ GN contains kinks of mass $m \propto e^{-1/g}$ belonging to the spinor representation and their bound states, the Majorana fermions of mass $m_b = \sqrt{2}m$, belonging to the fundamental representation of $SO(6)$ [10]. There are no higher-rank tensors present in $SO(6)$.

Due to the integrability, the number and energy-momenta of the particles in a scattering process are preserved, and the scattering amplitudes are factorized into two-particle S-matrices which, in turn, satisfy the Yang-Baxter equation. The Yang-Baxter equation along with symmetry, unitarity, and the crossing relation can determine the S-matrices of the GN model [10]. The S-matrix is a fundamental quantity in the integrable QFTs since it can be used to compute off-shell quantities, such as correlation functions, as well as on-shell quantities.

Computing correlation functions exactly is extremely difficult even for integrable QFTs away from criticality. The method in this paper is to use the form factor expansions including only a few leading contributions. Although this method does not provide exact correlation functions, it has been well-known that this expansion converges very rapidly and becomes accurate enough for practical purpose if one includes first few leading contributions. Another important feature of the form factor expansion is that only a finite number of terms are left in Fourier-transformed correlation functions like spectral functions of massive integrable QFTs, and they become exact due to the threshold effect of creating intermediate particle states. Therefore, the correlation function in momentum space becomes exact if one includes only a few leading terms in the low-frequency domain.

The form factors are matrix elements of a local operator between vacuum and on-shell particle states [11]. Consider a two-point correlation function

$$G^{\mathcal{O}}(x,t) = \langle 0 | T[\mathcal{O}(x,t)\mathcal{O}(0,0)] | 0 \rangle. \tag{23}$$

Using the completeness of on-shell particle states, one can insert the resolution of the identity between the two $\mathcal{O}$'s to obtain

$$G^{\mathcal{O}}(x,t) = \sum_I \left| \langle 0 | \mathcal{O} | I \rangle \right|^2 e^{itE_I + ixP_I}, \tag{24}$$

where $|I\rangle$ denotes on-shell states and the sum over I includes a sum and an integration over the particle contents and their energy-momenta. In two-dimensional relativistic QFTs, it is convenient to define the rapidity variable θ by

$$E = m\cosh\theta, \qquad P = \frac{m}{v}\sinh\theta. \tag{25}$$

E_I and P_I are the total energy and momentum of the on-shell state. The matrix element is the form factor. To be more specific, one can write the form factor as a function of the particle identities and their rapidities as follows:

$$\begin{aligned} & F^{\mathcal{O}}(\theta_1,\theta_2,\ldots,\theta_n)_{a_1,a_2,\ldots,a_n} \\ = \; & \langle 0|\mathcal{O}(0,0)|A_{a_1}(\theta_1),A_{a_2}(\theta_2),\ldots,A_{a_n}(\theta_n)\rangle. \end{aligned} \tag{26}$$

Here, $A_a(\theta)$ is a particle state of type a.

The form factors satisfy some functional relations with the S-matrices. In a few cases, generic form factors have been derived from these "axioms" [12]. In most integrable QFTs, it is straightforward to derive the form factors up to two-particle states from the exact S-matrices. We will compute the form factors up to two-particle states for the $SO(6)$ GN model in this paper.

IV. SPECTRAL FUNCTION

Being a Mott insulator, the Hubbard ladder away from half-filling becomes a conductor with gapless charge excitations. One can compute, for example, the current-current correlation function using the conformal field theory based on the Luttinger Hamiltonian. From Eq. (7), one can see that φ_1 is the gapless charge degree of freedom. The Hamiltonian can be written as

$$H_1 = \frac{\pi v_F}{2}\int dx\left[(1+M_1)\Pi_1^2 + \frac{1}{\pi^2}(1-M_1)(\partial_x\varphi_1)^2\right]. \tag{27}$$

This is the $c=1$ conformal field theory whose correlation functions can be easily computed. In particular, the two-point correlation function of the $U(1)$ current operator $(P=\pm)$,

$$J_P(x,t) = \partial_{x_P}\phi_{P1}, \qquad x_P = v_c t + Px, \tag{28}$$

can be obtained as

$$\langle J_P(x,t)J_P(0,0)\rangle = \frac{1}{(v_c t+Px)^2}. \tag{29}$$

Here, the renormalized velocity is given by

$$v_c = v_F\sqrt{1-M_1^2}. \tag{30}$$

It is more complicated to compute the single-particle spectral function. For this, we should first consider the single-particle Green's function. Since the electronic excitations around the Fermi point correspond to a separation of the charge degree of freedom and $SO(6)$ Gross-Neveu kinks, one can write the electron operators as follows:

$$c_{Pj\sigma} = \kappa_{Pj\sigma}e^{-i\phi_{P1}(x,t)/2}\psi^{\alpha}(x,t), \tag{31}$$

where ψ_α is the $SO(6)$ kink operator with spinor index α obtained by fermionizing the three bosons ϕ_{Pi} with $i=2,3,4$. The single-particle Green's function can be written as

$$\begin{aligned} G_{Pj}(k,\omega) \; &= \; \int dxdt e^{-ikx-i\omega t} \\ & \times \; \langle 0|T\left[c^\dagger_{Pj\sigma}(x,t)c_{Pj\sigma}(0,0)\right]|0\rangle, \end{aligned} \tag{32}$$

where T is the time ordering.

The correlation function becomes factorized into two parts:

$$\begin{aligned} & \langle 0|T\left[c^\dagger_{Pj\sigma}(x,t)c_{Pj\sigma}(0,0)\right]|0\rangle \\ = \; & \langle e^{i\phi_{P1}(x,t)/2}e^{-i\phi_{P1}(0,0)/2}\rangle \\ & \times \langle T(\kappa_{P\alpha}\psi^\alpha_P(x,t)\kappa_{P\bar\alpha}\psi^{\bar\alpha}_P(0,0))\rangle. \end{aligned} \tag{33}$$

The first factor can be computed easily from Eq. (27):

$$\langle e^{i\phi_{P1}(x,t)/2}e^{-i\phi_{P1}(0,0)/2}\rangle = \frac{1}{(v_c t+Px)^\nu}, \tag{34}$$

with

$$\nu = \frac{1}{4}\sqrt{\frac{1+M_1}{1-M_1}}. \tag{35}$$

To compute the second part, one should use the form factor expansion introduced in the previous section. The velocity parameter v' of the $SO(6)$ Gross-Neveu particles is given by the Luttinger parameter M:

$$v' = v_F\sqrt{1-M^2}. \tag{36}$$

If we expand the correlator up to the two-particle states, we get

$$\begin{aligned} & \langle T(\kappa_{P\alpha}\psi^\alpha_P(x,t>0)\kappa_{P\bar\alpha}\psi^{\bar\alpha}_P(0,0))\rangle \\ = \; & \int_{-\infty}^{\infty}\frac{d\theta}{2\pi}\left|\langle 0|\psi^\alpha_P(0,0)|A_\alpha(\theta)\rangle\right|^2 e^{iE_1(\theta)t+iP_1(\theta)x} \\ + \; & \sum_{a\beta}\int_{-\infty}^{\infty}\frac{d\theta_1 d\theta_2}{2(2\pi)^2}\left|\langle 0|\psi^\alpha_P|A_\beta(\theta_2)A_a(\theta_1)\rangle\right|^2 e^{iE_2 t+iP_2 x}, \end{aligned}$$

where E_2 and P_2 are the energy and the momentum of one kink and one fundamental fermion: namely,

$$\begin{aligned} E_1(\theta) \; &= \; m\cosh\theta, \quad v'P_1(\theta) = m\sinh\theta, \\ E_2(\theta_1,\theta_2) \; &= \; m_b\cosh\theta_1 + m\cosh\theta_2, \\ v'P_2(\theta_1,\theta_2) \; &= \; m_b\sinh\theta_1 + m\sinh\theta_2. \end{aligned}$$

The form factors can be computed from the exact S-matrices. The results are as follows: The one-particle form factor is

$$\langle 0|\psi^\alpha_\pm(0,0)|A_\beta(\theta_1)\rangle = A_F e^{\pm i\pi/4}e^{\pm\theta/2}C_{\alpha\beta}F(\theta) \tag{37}$$

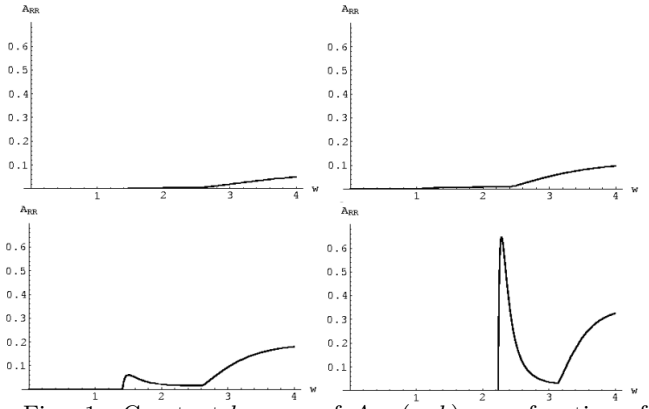

Fig. 1. Constant-k scans of $A_{RR}(\omega,k)$ as a function of ω/m. The figures are in the order of $k=-m$, $k=0$, $k=m$, and $k=2m$.

with

$$F(\theta) = \left(\frac{\Gamma\left(\frac{3}{4}+\frac{i\theta}{2\pi}\right)\Gamma\left(\frac{3}{2}-\frac{i\theta}{2\pi}\right)}{\Gamma\left(\frac{3}{4}-\frac{i\theta}{2\pi}\right)\Gamma\left(\frac{3}{2}+\frac{i\theta}{2\pi}\right)}\right)^{-1/2} \times \exp\left[-\int_0^\infty \frac{dx}{x}G(x)\sin^2\left(\frac{x\hat{\theta}}{2\pi}\right)\right],$$

$\hat{\theta}=\theta+i\pi$, and

$$G(x)=\frac{2\cosh(x/4)+e^{-5x/4}}{\sinh x\cosh(x/2)}. \tag{38}$$

The two-particle form factors are

$$\langle 0|\psi^\alpha_\pm(0,0)|A_\beta(\theta_2)A_a(\theta_1)\rangle = -A_F e^{\pm i\pi/4}(C\gamma^a)_{\alpha\beta} \times F_\pm(\theta_1,\theta_2)$$

with

$$F_\pm(\theta_1,\theta_2) = \frac{e^{\pm(\theta_1+\theta_2)/4}}{\cosh\theta_{12}} \times \exp\left[\int_0^\infty \frac{dx}{x}G(x)\sin^2\left(\frac{x\hat{\theta}_{12}}{2\pi}\right)\right]$$

and $\theta_{12}=\theta_1-\theta_2$. Here, C and γ^a are the charge conjugation and Dirac matrices, respectively. The constant A_F depends on the normalization, and we set it to be 1.

To derive the spectral functions, we should take the Fourier transform, Eq. (32), of the single-particle Green's function, Eq. (33). For simplicity, we derive the explicit expression for $P=+$ under the assumption that $M=M_1$ or $v_c=v'=1$. The case $v_c\neq v'$ can be similarly analyzed with more complicated algebra and shows a similar qualitative result. To take the Fourier transform, we change the integral variables to $x_\pm$. In terms of these variables, the correlation function of the charge sector depends only on x_+, and the exponential part is factorized $\exp(iK_+x_+ + iK_-x_-)$ where

$$K_\pm=\frac{1}{2}\left(E-\omega\pm(P-k)\right), \tag{39}$$

with E and P being either E_1 and P_1 for the one-particle state or E_2 and P_2 for the two-particle state. The x_- integral generates the Dirac delta function $\delta(K_-)$, and the x_+ integral becomes

$$\int dx_+ \frac{1}{x_+^\nu}e^{ix_+K_+} = A_\nu|K_+|^{1-\nu} \tag{40}$$

with $A_\nu=\sqrt{\frac{2}{\pi}}\Gamma(1-\nu)\sin\frac{\pi\nu}{2}$ for $0<\nu<1$. Using these results, one can compute each contribution to the spectral function separately. The one-particle contribution becomes ($\omega>k$)

$$G^{(1)}_{RR}(\omega,k)=\frac{A_\nu m^3}{2^{2-\nu}}\frac{(1-s^2/m^2)^{1-\nu}}{(\omega-k)^{3-\nu}}H(\theta_0) \tag{41}$$

where $s^2=\omega^2-k^2$, $me^{-\theta_0}=\omega-k$, and

$$H(z)=\exp\left[\int_0^\infty\frac{dx}{x}G(x)\left(1-\cosh x\cos\left(\frac{xz}{\pi}\right)\right)\right].$$

The two-particle contribution turns out to be

$$G^{(2)}_{RR}(\omega,k)=\frac{2^{3+\nu}A_\nu m^3}{(\omega-k)^{3-\nu}}\int_0^\infty dy\frac{y^2(\sqrt{3}y^2+1)}{(1+y^4)^2} \times\left[3+\sqrt{2}(y^2+y^{-2})-\frac{s^2}{m^2}\right]^{1-\nu}H(2\log y). \tag{42}$$

The spectral function is given by the imaginary part of the single-particle Green's function. For a nonvanishing spectral function, the variable s should be above the threshold. The threshold for an one-kink state is $s=m$ while that for a two-particle state (one-kink and one-bound state) is $s=(1+\sqrt{2})m$. The results for $s>m$ are

$$A^{(1)}_{RR}(\omega,k)=\frac{m^3A_\nu\sin\pi\nu}{2^{2-\nu}}\frac{(s^2/m^2-1)^{1-\nu}}{(\omega-k)^{3-\nu}}H(\theta_0), \tag{43}$$

and those for $s>(1+\sqrt{2})m$ are

$$A^{(2)}_{RR}(\omega,k)=\frac{2^{3+\nu}m^3A_\nu\sin\pi\nu}{(\omega-k)^{3-\nu}}\int_{y_1}^{y_2}dy\frac{y^2(\sqrt{3}y^2+1)}{(1+y^4)^2} \times\left[\frac{s^2}{m^2}-3-\sqrt{2}(y^2+y^{-2})\right]^{1-\nu}H(2\log y), \tag{44}$$

with y_1 and y_2 being two real solutions ($y_2>y_1>0$) of

$$3+\sqrt{2}(y^2+y^{-2})=\frac{s^2}{m^2}.$$

We can summarize that exact single-particle spectral function as

$$A_{RR} = A^{(1)}_{RR}, \quad 1<\frac{s}{m}<(1+\sqrt{2}),$$
$$A_{RR} = A^{(1)}_{RR}+A^{(2)}_{RR}, \quad (1+\sqrt{2})<\frac{s}{m}<(1+2\sqrt{2}).$$

We plot the result in Fig. 1 as a function of ω/m for various values of k for $\nu=0.25$. The threshold effects are clear from the figures.

In this paper we have shown a way to compute correlation functions based on integrable QFTs. Other physical quantities for the doped carbon nanotubes can be approached in this way. Also, carbon nanotubes have several other phases with different symmetries. It would be interesting to study these phases using QFT methods.

ACKNOWLEDGMENTS

We thank Dr. Hyunwoo Lee for varuable discussions. This work is supported in part by Korean Research Foundation 2001-015-D00071, Ministry of Science and Technology 99-N6-01-01-A-5, and Korean Science and Engineering Foundation 1999-2-112-001-5.

REFERENCES

[1] E. Dagotto, Science **271**, 618 (1996); T. Ebbesen, Phys. Today **49**, 26 (1996).

[2] Y. Kim, C. Park and K. J. Chang, J. Korean Phys. Soc. **37**, 85 (2000); J. Wei and J. Hong, J. Korean Phys. Soc. **38**, 602 (2001).

[3] Y. Krotov, D. H. Lee and G. Louie, Phys. Rev. Lett. **78**, 4245 (1997).

[4] L. Balents and M. P. A. Fisher, Phys. Rev. B **55**, 11973 (1997).

[5] D. Gross and A. Neveu, Phys. Rev. D **10**, 3235 (1974).

[6] H. Lin, L. Balents and M. Fisher, `cond-mat/9801285`.

[7] R. Konik and A. Ludwig, `cond-mat/9810332`.

[8] H. J. Schulz, `cond-mat/9808167`.

[9] A. Zamolodchikov and Al. Zamolodchikov, Annals. Phys. **120**, 253 (1979).

[10] M. Karowski and H.Thun, Nucl. Phys. B **190**, 61 (1981).

[11] M. Karowski and P. Weisz, Nucl. Phys. B **139**, 455 (1978).

[12] F. A. Smirnov, *Form Factors in Completely Integrable Models of Quantum Field Theory* (World Scientific, Singapore, 1992).