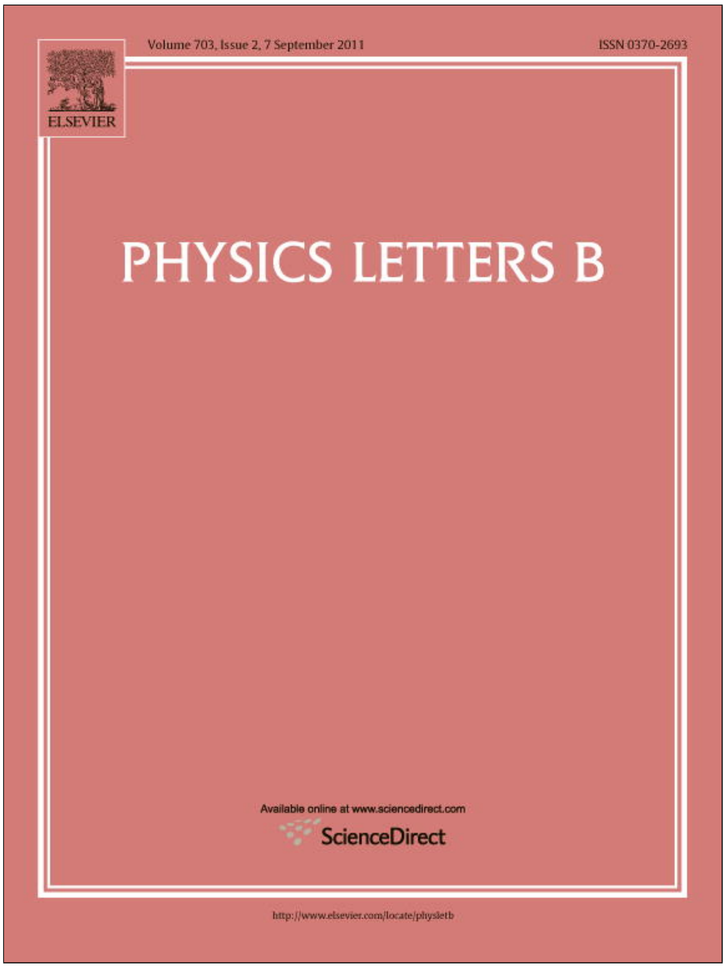
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Finite-size giant magnons on $AdS_4 \times CP^3_\gamma$

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ABSTRACT

We investigate finite-size giant magnons propagating on γ-deformed $AdS_4 \times CP^3_\gamma$ type IIA string theory background, dual to one parameter deformation of the $\mathcal{N}=6$ super Chern–Simons-matter theory. Analyzing the finite-size effect on the dispersion relation, we find that it is modified compared to the undeformed case, acquiring γ dependence.

1. Introduction

Investigations on AdS/CFT duality [1] for theories with reduced or without supersymmetry are important not only conceptually, but also for describing realistic physics. An example of such correspondence between gauge and string theory models with reduced supersymmetry is provided by an exactly marginal deformation of $\mathcal{N}=4$ super Yang–Mills theory [2] and string theory on a β-deformed $AdS_5 \times S^5$ background suggested in [3]. When $\beta \equiv \gamma$ is real, the deformed background can be obtained from $AdS_5 \times S^5$ by the so-called TsT transformation. It includes T-duality on one angle variable, a shift of another isometry variable, then a second T-duality on the first angle [3,4].

Another interesting example is the duality between the γ-deformed $AdS_4 \times CP^3_\gamma$ type IIA string theory and one parameter deformation of the ABJM theory [5], i.e. $\mathcal{N}=6$ super Chern–Simons-matter theory in three dimensions. The resulting theory has $\mathcal{N}=2$ supersymmetry and the modified superpotential is [6]

$$W_\gamma \propto \mathrm{Tr}\left(e^{-i\pi\gamma/2} A_1 B_1 A_2 B_2 - e^{i\pi\gamma/2} A_1 B_2 A_2 B_1\right). \tag{1.1}$$

Here the chiral superfields A_i, B_i $(i=1,2)$ represent the matter part of the theory. As in the $\mathcal{N}=4$ super Yang–Mills case, the marginality of the deformation translates into the fact that AdS_4 part of the background is untouched. Taking into account that CP^3 has three isometric coordinates, one can consider a chain of three TsT transformations. The result is a regular three-parameter deformation of $AdS_4 \times CP^3$ string background, dual to a non-supersymmetric deformation of ABJM theory, which reduces to the supersymmetric one by putting $\gamma_1 = \gamma_2 = 0$ and $\gamma_3 = \gamma$ [6].

The dispersion relation for the giant magnon [7] in the γ-deformed $AdS_4 \times CP^3_\gamma$ background, carrying two nonzero angular momenta, has been found in [8]. Here we are interested in obtaining the finite-size correction to it. To this end, in Section 2 we introduce the γ-deformed background, consider strings on the $R_t \times RP^3_\gamma$ subspace of $AdS_4 \times CP^3_\gamma$, and find the exact expressions for the conserved charges and the angular differences. In Section 3 we perform the necessary expansions, and derive the leading corrections to the dispersion relations of giant magnons with one and two angular momenta. In Section 4 we conclude with some remarks.

2. Exact results

Let us first write down the deformed background. It is given by [6][2]

$$ds^2_{IIA} = R^2\left(\frac{1}{4} ds^2_{AdS_4} + ds^2_{CP^3_\gamma}\right),$$

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[2] There are also nontrivial dilaton and fluxes F_2, F_4, but since the fundamental string does not interact with them at the classical level, we do not need to know the corresponding expressions.

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$$ds^2_{CP^3_\gamma} = d\psi^2 + G\sin^2\psi\cos^2\psi\left(\frac{1}{2}\cos\theta_1\,d\phi_1 - \frac{1}{2}\cos\theta_2\,d\phi_2 + d\phi_3\right)^2 + \frac{1}{4}\cos^2\psi\left(d\theta_1^2 + G\sin^2\theta_1\,d\phi_1^2\right) + \frac{1}{4}\sin^2\psi\left(d\theta_2^2 + G\sin^2\theta_2\,d\phi_2^2\right) + \tilde{\gamma}G\sin^4\psi\cos^4\psi\sin^2\theta_1\sin^2\theta_2\,d\phi_3^2,$$

$$B_2 = -R^2\tilde{\gamma}G\sin^2\psi\cos^2\psi \times\left[\frac{1}{2}\cos^2\psi\sin^2\theta_1\cos\theta_2\,d\phi_3\wedge d\phi_1 + \frac{1}{2}\sin^2\psi\sin^2\theta_2\cos\theta_1\,d\phi_3\wedge d\phi_2 + \frac{1}{4}\left(\sin^2\theta_1\sin^2\theta_2 + \cos^2\psi\sin^2\theta_1\cos^2\theta_2 + \sin^2\psi\sin^2\theta_2\cos^2\theta_1\right)d\phi_1\wedge d\phi_2\right],$$

where

$$G^{-1} = 1 + \tilde{\gamma}^2\sin^2\psi\cos^2\psi\left(\sin^2\theta_1\sin^2\theta_2 + \cos^2\psi\sin^2\theta_1\cos^2\theta_2 + \sin^2\psi\sin^2\theta_2\cos^2\theta_1\right).$$

The deformation parameter $\tilde{\gamma}$ above is given by $\tilde{\gamma} = \frac{R^2}{4}\gamma$, where γ appears in the dual field theory superpotential (1.1).

2.1. String solutions

In our considerations we will use conformal gauge, in which the string Lagrangian and Virasoro constraints have the form

$$\mathcal{L}_s = \frac{T}{2}(G_{00} - G_{11} + 2B_{01}), \tag{2.1}$$

$$G_{00} + G_{11} = 0, \qquad G_{01} = 0. \tag{2.2}$$

Here

$$G_{mn} = g_{MN}\partial_m X^M\partial_n X^N, \qquad B_{mn} = b_{MN}\partial_m X^M\partial_n X^N,$$
$$\partial_m = \partial/\partial\xi^m, \quad m,n = (0,1), \qquad \left(\xi^0,\xi^1\right) = (\tau,\sigma), \quad M,N = (0,1,\ldots,9),$$

are the fields induced on the string worldsheet.

Further on, we restrict our attention to the $R_t\times RP^3_\gamma$ subspace of $AdS_4\times CP^3_\gamma$, where $\theta_1 = \theta_2 = \pi/2$, $\phi_3 = 0$, and

$$ds^2 = R^2\left(-\frac{1}{4}dt^2 + d\psi^2 + \frac{G}{4}\cos^2\psi\,d\phi_1^2 + \frac{G}{4}\sin^2\psi\,d\phi_2^2\right),$$
$$B_2 = b_{\phi_1\phi_2}\,d\phi_1\wedge d\phi_2 = -\frac{R^2}{4}\tilde{\gamma}G\sin^2\psi\cos^2\psi\,d\phi_1\wedge d\phi_2,$$
$$G^{-1} = 1 + \tilde{\gamma}^2\sin^2\psi\cos^2\psi.$$

To find the string solutions we are interested in, we use the ansatz ($j = 1,2$)

$$t(\tau,\sigma) = \kappa\tau, \qquad \psi(\tau,\sigma) = \psi(\xi), \qquad \phi_j(\tau,\sigma) = \omega_j\tau + f_j(\xi),$$
$$\xi = \alpha\sigma + \beta\tau, \quad \kappa,\omega_j,\alpha,\beta = \text{constants}. \tag{2.3}$$

Then the string Lagrangian (2.1) becomes (prime is used for $d/d\xi$)

$$\mathcal{L}_s = -\frac{TR^2}{2}(\alpha^2-\beta^2)\left[\psi'^2 + \frac{G}{4}\cos^2\psi\left(f_1' - \frac{\beta\omega_1}{\alpha^2-\beta^2}\right)^2 + \frac{G}{4}\sin^2\psi\left(f_2' - \frac{\beta\omega_2}{\alpha^2-\beta^2}\right)^2 - \frac{G\alpha^2}{4(\alpha^2-\beta^2)^2}\left(\omega_1^2\cos^2\psi + \omega_2^2\sin^2\psi\right) + \frac{\alpha\tilde{\gamma}G}{2}\sin^2\psi\cos^2\psi\frac{\omega_1 f_2' - \omega_2 f_1'}{\alpha^2-\beta^2}\right], \tag{2.4}$$

while the constraints (2.2) acquire the form

$$\psi'^2 + \frac{G}{4}\cos^2\psi\left(f_1'^2 + \frac{2\beta\omega_1}{\alpha^2+\beta^2}f_1' + \frac{\omega_1^2}{\alpha^2+\beta^2}\right) + \frac{G}{4}\sin^2\psi\left(f_2'^2 + \frac{2\beta\omega_2}{\alpha^2+\beta^2}f_2' + \frac{\omega_2^2}{\alpha^2+\beta^2}\right) = \frac{\kappa^2/4}{\alpha^2+\beta^2},$$
$$\psi'^2 + \frac{G}{4}\cos^2\psi\left(f_1'^2 + \frac{\omega_1}{\beta}f_1'\right) + \frac{G}{4}\sin^2\psi\left(f_2'^2 + \frac{\omega_2}{\beta}f_2'\right) = 0. \tag{2.5}$$

The equations of motion for $f_j(\xi)$ following from (2.4) can be integrated once to give

$$f_1' = \frac{1}{\alpha^2-\beta^2}\left[\frac{C_1}{\cos^2\psi} + \beta\omega_1 + \tilde{\gamma}(\alpha\omega_2 + \tilde{\gamma}C_1)\sin^2\psi\right],$$
$$f_2' = \frac{1}{\alpha^2-\beta^2}\left[\frac{C_2}{\sin^2\psi} + \beta\omega_2 - \tilde{\gamma}(\alpha\omega_1 - \tilde{\gamma}C_2)\cos^2\psi\right], \tag{2.6}$$

where C_j are constants. Replacing (2.6) into (2.5), one can rewrite the Virasoro constraints as

$$\psi'^2 = \frac{1}{4(\alpha^2-\beta^2)^2}\left[(\alpha^2+\beta^2)\kappa^2 - \frac{C_1^2}{\cos^2\psi} - \frac{C_2^2}{\sin^2\psi} - (\alpha\omega_1 - \tilde{\gamma}C_2)^2\cos^2\psi - (\alpha\omega_2 + \tilde{\gamma}C_1)^2\sin^2\psi\right], \tag{2.7}$$

$$\omega_1 C_1 + \omega_2 C_2 + \beta\kappa^2 = 0. \tag{2.8}$$

Let us point out that (2.7) is the first integral of the equation of motion for ψ. Integrating (2.6) and (2.7), one can find string solutions with very different properties. All of them are related to solutions of the complex sine-Gordon integrable model in an explicit way [10]. Particular examples are (dyonic) giant magnons and single-spike strings.

2.2. Conserved quantities and angular differences

In the case at hand, the background metric does not depend on t and ϕ_j. The corresponding conserved quantities are the string energy E_s and two angular momenta J_j, given by

$$E_s = -\int d\sigma\, \frac{\partial\mathcal{L}_s}{\partial(\partial_0 t)}, \qquad J_j = \int d\sigma\, \frac{\partial\mathcal{L}_s}{\partial(\partial_0 \phi_j)}. \tag{2.9}$$

On the ansatz (2.3), E_s and J_j defined above take the form

$$E_s = \frac{TR^2}{4}\frac{\kappa}{\alpha}\int d\xi,$$
$$J_1 = \frac{TR^2}{4}\frac{1}{\alpha^2-\beta^2}\int d\xi\left[\frac{\beta}{\alpha}C_1 + (\alpha\omega_1 - \tilde{\gamma}C_2)\cos^2\psi\right],$$
$$J_2 = \frac{TR^2}{4}\frac{1}{\alpha^2-\beta^2}\int d\xi\left[\frac{\beta}{\alpha}C_2 + (\alpha\omega_2 + \tilde{\gamma}C_1)\sin^2\psi\right]. \tag{2.10}$$

Let us remind that the relation between the string tension T and the 't Hooft coupling constant λ for the $\mathcal{N}=6$ super Chern–Simons-matter theory is given by

$$TR^2 = 2\sqrt{2\lambda}.$$

If we introduce the variable

$$\chi = \cos^2\psi,$$

and use (2.8), the first integral (2.7) can be rewritten as

$$\chi'^2 = \frac{\Omega_2^2(1-u^2)}{\alpha^2(1-v^2)^2}(\chi_p - \chi)(\chi - \chi_m)(\chi - \chi_n),$$

where

$$\chi_p + \chi_m + \chi_n = \frac{2-(1+v^2)W - u^2}{1-u^2},$$
$$\chi_p\chi_m + \chi_p\chi_n + \chi_m\chi_n = \frac{1-(1+v^2)W + (vW - uK)^2 - K^2}{1-u^2},$$
$$\chi_p\chi_m\chi_n = -\frac{K^2}{1-u^2}, \tag{2.11}$$

and

$$v = -\frac{\beta}{\alpha}, \qquad u = \frac{\Omega_1}{\Omega_2}, \qquad W = \left(\frac{\kappa}{\Omega_2}\right)^2, \qquad K = \frac{C_1}{\alpha\Omega_2},$$
$$\Omega_1 = \omega_1\left(1 - \tilde{\gamma}\frac{C_2}{\alpha\omega_1}\right), \qquad \Omega_2 = \omega_2\left(1 + \tilde{\gamma}\frac{C_1}{\alpha\omega_2}\right).$$

We are interested in the case

$$0 < \chi_m < \chi < \chi_p < 1, \qquad \chi_n < 0,$$

which corresponds to the finite-size giant magnons.

In terms of the newly introduced variables, the conserved quantities (2.10) and the angular differences

$$p_1 \equiv \Delta\phi_1 = \phi_1(r) - \phi_1(-r), \qquad p_2 \equiv \Delta\phi_2 = \phi_2(r) - \phi_2(-r), \tag{2.12}$$

transform to

$$\mathcal{E} \equiv \frac{E_s}{TR^2} = \frac{(1-v^2)\sqrt{W}}{\sqrt{1-u^2}} \frac{\mathbf{K}(1-\epsilon)}{\sqrt{\chi_p - \chi_n}}, \tag{2.13}$$

$$\mathcal{J}_1 \equiv \frac{J_1}{TR^2} = \frac{1}{\sqrt{1-u^2}}\left[\frac{u\chi_n - vK}{\sqrt{\chi_p-\chi_n}}\mathbf{K}(1-\epsilon) + u\sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon)\right], \tag{2.14}$$

$$\mathcal{J}_2 \equiv \frac{J_2}{TR^2} = \frac{1}{\sqrt{1-u^2}}\left[\frac{1-\chi_n - v(vW-uK)}{\sqrt{\chi_p-\chi_n}}\mathbf{K}(1-\epsilon) - \sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon)\right], \tag{2.15}$$

$$p_1 = \frac{4}{\sqrt{1-u^2}}\left\{\frac{K}{\chi_p\sqrt{\chi_p-\chi_n}}\Pi\left(1-\frac{\chi_m}{\chi_p}\middle|1-\epsilon\right) - \left[uv + \tilde\gamma v(vW-uK) - \tilde\gamma(1-\chi_n)\right]\frac{\mathbf{K}(1-\epsilon)}{\sqrt{\chi_p-\chi_n}} - \tilde\gamma\sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon)\right\}, \tag{2.16}$$

$$p_2 = \frac{4}{\sqrt{1-u^2}}\left\{\frac{vW-uK}{(1-\chi_p)\sqrt{\chi_p-\chi_n}}\Pi\left(-\frac{\chi_p-\chi_m}{1-\chi_p}\middle|1-\epsilon\right) - \left[v(1-\tilde\gamma K) + \tilde\gamma u\chi_n\right]\frac{\mathbf{K}(1-\epsilon)}{\sqrt{\chi_p-\chi_n}} - \tilde\gamma u\sqrt{\chi_p-\chi_n}\mathbf{E}(1-\epsilon)\right\}, \tag{2.17}$$

where we have used the formulas for the elliptic integrals given in Appendix A, and ϵ is given by

$$\epsilon = \frac{\chi_m - \chi_n}{\chi_p - \chi_n}. \tag{2.18}$$

From (2.13)–(2.15) one can see that the conserved charges are not affected by the γ-deformation as it should be. Only the angular differences are shifted.

Further on, we will consider the case when $\mathcal{E}$, $\mathcal{J}_2$ and p_1 are large, while $\mathcal{E} - \mathcal{J}_2$, $\mathcal{J}_1$ and p_2 are finite. To this end, we will introduce appropriate expansions.

3. Expansions

In order to find the leading finite-size correction to the energy–charge relation, we have to consider the limit $\epsilon \to 0$ in (2.11), (2.13)–(2.15), and (2.18). The behavior of the complete elliptic integrals in this limit is given in Appendix A. Taking this behavior into account, we will use the following ansatz for the parameters $(\chi_p, \chi_m, \chi_n, v, u, W, K)$ in the solution

$$\begin{aligned}
\chi_p &= \chi_{p0} + (\chi_{p1} + \chi_{p2}\log(\epsilon))\epsilon,\\
\chi_m &= \chi_{m0} + (\chi_{m1} + \chi_{m2}\log(\epsilon))\epsilon,\\
\chi_n &= \chi_{n0} + (\chi_{n1} + \chi_{n2}\log(\epsilon))\epsilon,\\
v &= v_0 + (v_1 + v_2\log(\epsilon))\epsilon,\\
u &= u_0 + (u_1 + u_2\log(\epsilon))\epsilon,\\
W &= W_0 + (W_1 + W_2\log(\epsilon))\epsilon,\\
K &= K_0 + (K_1 + K_2\log(\epsilon))\epsilon.
\end{aligned} \tag{3.1}$$

A few comments are in order. To be able to reproduce the dispersion relation for the infinite-size giant magnons, we set

$$\chi_{m0} = \chi_{n0} = K_0 = 0, \qquad W_0 = 1. \tag{3.2}$$

Also to reproduce the undeformed case [9] in the $\tilde\gamma \to 0$ limit, we need to fix

$$\chi_{m2} = \chi_{n2} = W_2 = K_2 = 0. \tag{3.3}$$

Replacing (3.1) into (2.11) and (2.18), one finds six equations for the coefficients in the expansions of χ_p, χ_m, χ_n and W. They are solved by

$$\begin{aligned}
\chi_{p0} &= 1 - \frac{v_0^2}{1-u_0^2},\\
\chi_{p1} &= \frac{v_0}{(1-v_0^2)(1-u_0^2)(1-v_0^2-u_0^2)}\Big\{-2v_0u_0(1-v_0^2)(1-v_0^2-u_0^2)u_1\\
&\quad + 2(1-u_0^2)(1-v_0^2-u_0^2)\left[K_1u_0(1+v_0^2) - (1-v_0^2)v_1\right]\\
&\quad + v_0(1-v_0^2-2u_0^2)\sqrt{(1-u_0^2-v_0^2)^4 - 4K_1^2(1-u_0^2)^2(1-u_0^2-v_0^2)}\Big\},
\end{aligned}$$

$$\chi_{p2} = -2v_0 \frac{v_2 + (v_0 u_2 - u_0 v_2)u_0}{(1-u_0^2)^2},$$

$$\chi_{m1} = \frac{u_0^4 - 2u_0^2(1-v_0^2) + (1-v_0^2)^2 + \sqrt{(1-u_0^2-v_0^2)^4 - 4K_1^2(1-u_0^2)^2(1-u_0^2-v_0^2)}}{2(1-u_0^2)(1-v_0^2-u_0^2)},$$

$$\chi_{n1} = -\frac{u_0^4 - 2u_0^2(1-v_0^2) + (1-v_0^2)^2 - \sqrt{(1-u_0^2-v_0^2)^4 - 4K_1^2(1-u_0^2)^2(1-u_0^2-v_0^2)}}{2(1-u_0^2)(1-v_0^2-u_0^2)},$$

$$W_1 = -\frac{2K_1 u_0 v_0 (1-u_0^2) + \sqrt{(1-u_0^2-v_0^2)^4 - 4K_1^2(1-u_0^2)^2(1-u_0^2-v_0^2)}}{(1-u_0^2)(1-v_0^2)}. \tag{3.4}$$

As a next step, we impose the conditions for $\mathcal{J}_1$, p_2 to be independent of ϵ. By expanding RHS of (2.14), (2.17) in ϵ, one gets

$$\mathcal{J}_1 = \frac{u_0\sqrt{1-v_0^2-u_0^2}}{1-u_0^2}, \tag{3.5}$$

$$p_2 = 2\arcsin\left(\frac{2v_0\sqrt{1-v_0^2-u_0^2}}{1-u_0^2}\right) - 4\tilde{\gamma} u_0 \frac{\sqrt{1-v_0^2-u_0^2}}{1-u_0^2}, \tag{3.6}$$

along with four more equations from the coefficients of ϵ and $\epsilon \log \epsilon$. The equalities (3.5), (3.6) lead to

$$v_0 = \frac{\sin\Psi}{2\sqrt{\mathcal{J}_1^2 + \sin^2(\Psi/2)}}, \qquad u_0 = \frac{\mathcal{J}_1}{\sqrt{\mathcal{J}_1^2 + \sin^2(\Psi/2)}}, \qquad p_2 = 2(\Psi - 2\tilde{\gamma}\mathcal{J}_1), \tag{3.7}$$

where the angle Ψ is defined as

$$\Psi = \arcsin\left(\frac{2v_0\sqrt{1-v_0^2-u_0^2}}{1-u_0^2}\right).$$

After the replacement of (3.4) into the remaining four equations, they can be solved with respect to v_1, v_2, u_1, u_2, leading to the following form of the dispersion relation in the considered approximation

$$\mathcal{E} - \mathcal{J}_2 = \frac{\sqrt{1-v_0^2-u_0^2}}{1-u_0^2} - \frac{1}{4}\frac{\sqrt{(1-v_0^2-u_0^2)^3 - 4K_1^2(1-u_0^2)^2}}{1-u_0^2}\epsilon. \tag{3.8}$$

To the leading order, the expansion for $\mathcal{J}_2$ gives

$$\epsilon = 16\exp\left[-\frac{2}{1-v_0^2}\left(1 - \frac{v_0^2}{1-u_0^2} + \mathcal{J}_2\sqrt{1-v_0^2-u_0^2}\right)\right]. \tag{3.9}$$

By using (3.7) and (3.9), (3.8) can be rewritten as

$$\mathcal{E} - \mathcal{J}_2 = \sqrt{\mathcal{J}_1^2 + \sin^2(\Psi/2)} - 4\sqrt{\frac{\sin^8(\Psi/2)}{\mathcal{J}_1^2 + \sin^2(\Psi/2)} - 4K_1^2 \exp\left[-\frac{2\left(\mathcal{J}_2 + \sqrt{\mathcal{J}_1^2 + \sin^2(\Psi/2)}\right)\sqrt{\mathcal{J}_1^2 + \sin^2(\Psi/2)}\sin^2(\Psi/2)}{\mathcal{J}_1^2 + \sin^4(\Psi/2)}\right]}. \tag{3.10}$$

The parameter K_1 in (3.10) can be related to the angular difference p_1. To see that, let us consider the leading order in the ϵ-expansion for it:

$$p_1 = \frac{4K_1 \arctan\sqrt{\frac{\chi_{p0}}{\chi_{m1}} - 1}}{\sqrt{(1-u_0^2)\chi_{p0}\chi_{m1}(\chi_{p0} - \chi_{m1})}} - \frac{2}{\sqrt{(1-u_0^2)\chi_{p0}}}\left[u_0 v_0 \log(16) + \tilde{\gamma}\left(2\chi_{p0} - \left(1-v_0^2\right)\log(16)\right)\right]$$
$$+ \frac{2}{\sqrt{(1-u_0^2)\chi_{p0}}}\left[u_0 v_0 - \tilde{\gamma}\left(1-v_0^2\right)\right]\log(\epsilon). \tag{3.11}$$

So, it is natural to introduce the angle Φ as

$$\frac{\Phi}{2} = \arctan\sqrt{\frac{\chi_{p0}}{\chi_{m1}} - 1}. \tag{3.12}$$

On the solution for the other parameters this gives

$$K_1 = \frac{(1 - v_0^2 - u_0^2)^{3/2}}{2(1 - u_0^2)} \sin(\Phi) = \frac{\sin^4(\Psi/2)}{2\sqrt{\mathcal{J}_1^2 + \sin^2(\Psi/2)}} \sin(\Phi). \tag{3.13}$$

As a result, the relation (3.11) between the angles p_1 and Φ becomes

$$\Phi = \frac{p_1}{2} - \left(2\tilde{\gamma} - \mathcal{J}_1 \frac{\sin\Psi}{\mathcal{J}_1^2 + \sin^4(\Psi/2)}\right)\mathcal{J}_2 + \mathcal{J}_1 \frac{\sin\Psi\sqrt{\mathcal{J}_1^2 + \sin^2(\Psi/2)}}{\mathcal{J}_1^2 + \sin^4(\Psi/2)}, \tag{3.14}$$

where due to the periodicity condition we should set

$$p_1 = 2\pi n_1, \quad n_1 \in \mathbb{Z}.$$

Finally, in view of (3.13), the dispersion relation (3.10) for the dyonic giant magnons acquires the form

$$\mathcal{E} - \mathcal{J}_2 = \sqrt{\mathcal{J}_1^2 + \sin^2(\Psi/2)} - \frac{4\sin^4(\Psi/2)}{\sqrt{\mathcal{J}_1^2 + \sin^2(\Psi/2)}} \cos\Phi \exp\left[-\frac{2(\mathcal{J}_2 + \sqrt{\mathcal{J}_1^2 + \sin^2(\Psi/2)})\sqrt{\mathcal{J}_1^2 + \sin^2(\Psi/2)}\sin^2(\Psi/2)}{\mathcal{J}_1^2 + \sin^4(\Psi/2)}\right]. \tag{3.15}$$

Based on the Lüscher μ-term formula for the undeformed case [11], we propose to identify the angle $\Psi (= \frac{p_2}{2} + 2\tilde{\gamma}\mathcal{J}_1)$ with the momentum p of the magnon excitations in the dual spin chain. Now the angular differences are shifted and the leading finite-size correction to the dispersion relation is modified by $\cos\Phi$ compared with the undeformed $AdS_4 \times CP^3$. Also notice that this result is consistent with [8] in the infinite J limit after redefining the momentum p appropriately.

Let us point out that (3.15) has the same form as the dispersion relation for dyonic giant magnons on $R_t \times S^3_\gamma$ subspace of the γ-deformed $AdS_5 \times S^5$ [12].[3] Actually, the two energy–charge relations coincide after appropriate normalization of the charges and after exchange of the indices 1 and 2. The only remaining difference is in the first terms in the expressions for the angle Φ:

$$R_t \times RP^3_\gamma: \quad \Phi = \frac{p_1}{2} + \cdots,$$
$$R_t \times S^3_\gamma: \quad \Phi = p_2 + \cdots.$$

All of the above results simplify a lot when one consider giant magnons with one angular momentum, i.e. $\mathcal{J}_1 = 0$. In particular, the energy–charge relation (3.15) reduces to

$$\mathcal{E} - \mathcal{J}_2 = \sin\frac{p}{2}\left[1 - 4\sin^2\frac{p}{2}\cos(\pi n_1 - 2\tilde{\gamma}\mathcal{J}_2)e^{-2-2\mathcal{J}_2\csc\frac{p}{2}}\right]. \tag{3.16}$$

We want to point out that our result is different from [16] which has extra $\cos^3(p/4)$ in the denominator of the phase Φ.

4. Concluding remarks

In this Letter we considered string configurations on the $R_t \times RP^3_\gamma$ subspace of $AdS_4 \times CP^3_\gamma$. Imposing appropriate conditions on the parameters involved, we restrict ourselves to string solutions, which describe the finite-size giant magnons with one and two angular momenta. Taking the limit in which the modulus of the elliptic integrals approaches one from below, we found the leading corrections to the dispersion relations. The obtained results are relevant for comparison with the dual field theory, which in the case at hand is the one parameter γ-deformation of the $\mathcal{N} = 6$ super Chern–Simons-matter theory in three space–time dimensions.

It would be interesting to understand how to reproduce the dispersion relation (3.15) by using Lüscher's approach [13]. The dispersion relation has a specific $\tilde{\gamma}$-dependence for finite $\mathcal{J}_2$, and it is not quite clear how such a dependence follows from the S-matrix approach. To this end, we need a generalization of the Lüscher's formulas for the case of nontrivial *twisted* boundary conditions.

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Appendix A. Elliptic integrals and ϵ-expansions

The elliptic integrals appearing in the main text are given by

$$\int_{\chi_m}^{\chi_p} \frac{d\chi}{\sqrt{(\chi_p - \chi)(\chi - \chi_m)(\chi - \chi_n)}} = \frac{2}{\sqrt{\chi_p - \chi_n}}\mathbf{K}(1 - \epsilon),$$

$$\int_{\chi_m}^{\chi_p} \frac{\chi d\chi}{\sqrt{(\chi_p - \chi)(\chi - \chi_m)(\chi - \chi_n)}} = \frac{2\chi_n}{\sqrt{\chi_p - \chi_n}}\mathbf{K}(1 - \epsilon) + 2\sqrt{\chi_p - \chi_n}\mathbf{E}(1 - \epsilon),$$

[3] In [12], we left a numerical factor Λ in the definition of the angle Φ undetermined. Actually, $\Lambda = 1$. Also, the coefficient $1/2$ in front of the last term of (4.5) in [12] is a misprint.

$$\int_{\chi_m}^{\chi_p} \frac{d\chi}{\chi\sqrt{(\chi_p-\chi)(\chi-\chi_m)(\chi-\chi_n)}} = \frac{2}{\chi_p\sqrt{\chi_p-\chi_n}}\Pi\left(1-\frac{\chi_m}{\chi_p}\middle|1-\epsilon\right),$$

$$\int_{\chi_m}^{\chi_p} \frac{d\chi}{(1-\chi)\sqrt{(\chi_p-\chi)(\chi-\chi_m)(\chi-\chi_n)}} = \frac{2}{(1-\chi_p)\sqrt{\chi_p-\chi_n}}\Pi\left(-\frac{\chi_p-\chi_m}{1-\chi_p}\middle|1-\epsilon\right),$$

where

$$\epsilon = \frac{\chi_m-\chi_n}{\chi_p-\chi_n}.$$

We use the following expansions for the complete elliptic integrals [14]

$$\mathbf{K}(1-\epsilon) = -\frac{1}{2}\log\left(\frac{\epsilon}{16}\right) - \frac{1}{4}\left(1+\frac{1}{2}\log\left(\frac{\epsilon}{16}\right)\right)\epsilon + \cdots,$$

$$\mathbf{E}(1-\epsilon) = 1 - \frac{1}{4}\left(1+\log\left(\frac{\epsilon}{16}\right)\right)\epsilon + \cdots,$$

$$\Pi(-n|1-\epsilon) = \frac{2\sqrt{n}\arctan(\sqrt{n}) - \log(\frac{\epsilon}{16})}{2(1+n)} - \frac{2-4\sqrt{n}\arctan(\sqrt{n}) + (1-n)\log(\frac{\epsilon}{16})}{8(1+n)^2}\epsilon + \cdots, \quad n>0.$$

We use also the equality [15]

$$\Pi(\nu|m) = \frac{q_1}{q}\Pi(\nu_1|m) - \frac{m}{q\sqrt{-\nu\nu_1}}\mathbf{K}(m),$$

where

$$q = \sqrt{(1-\nu)\left(1-\frac{m}{\nu}\right)}, \qquad q_1 = \sqrt{(1-\nu_1)\left(1-\frac{m}{\nu_1}\right)},$$

$$\nu = \frac{\nu_1 - m}{\nu_1 - 1}, \quad \nu_1 < 0, \quad m < \nu < 1.$$

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